Set Cover Problem

Given:

A Universe U of n elements

A collection of subsets of U,

$S = \{S_1, S_2 \ldots\ldots S_k\}$

A cost function $c: S \rightarrow Q^+$

To Find: A minimum cost subcollection of S that covers all elements of U.

Some Notations and Definitions

C : set of elements already covered at the beginning of an iteration.

Cost Effectiveness of S: Average cost at which it covers new elements, i.e. $c(S) / |S-C|$.

Price of an element: Average cost at which it is covered, i.e. $price(e) = c(S) / |S-C|$.

Greedy Set Cover Algorithm

1. $C \leftarrow \Phi$
2. While $C \neq U$ do
   - Find the most cost-effective set in the current iteration, say S.
   - Let $\alpha = \frac{cost(S)}{S - C}$,i.e., the cost effectiveness of S
   - Pick S, and for each e in S-C, price(e)= α
   - $C \leftarrow C \cup S$
5. Output the picked sets.

The greedy algorithm is an O(log n) factor approximation algorithm: Proof Skipped.

Any Questions….

Thank you!